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Probabilistic Approach for Integrated Structural Control Design

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Introduction

DURING the development phase of a new engineering project, the design requirements and the project variables are usually defined in a deterministic sense, and, for robustness considerations, the uncertainty of the parameters is considered to occur around their design value by means of simplified models. Sufficient conditions must then be satisfied to achieve confidence in the quality of the final design. To meet the requirement constraints, a control strategy can be integrated within the structural design process so that the design of the structure and that of the control system are developed in parallel. The use of a classical robust control system design may be properly employed in the case where the problem at hand is convex and sufficient conditions can be met. In this case, however, the real distribution of the random variables involved in the design is not considered.

As an alternative, we suggest considering the problem of integrating the control and structure design within an appropriate probabilistic framework in order to take into account, with a high degree of accuracy, the project constraints and the system uncertainties. This approach implies the possibility of modeling the design parameters as random variables in order to guarantee that a given probability of satisfying the design requirements may be obtained. In many cases, the design through probabilistic constraints is performed with the hypothesis of normal distribution of the involved variables,[1] even if this hypothesis may be too strong an assumption. A better model is possible using some theoretical results that were first obtained in the field of structural reliability in civil and offshore engineering. These results led to the definition of the so-called classical structural reliability methodologies. These methodologies allow one to properly take into account the correct probability distribution function of the design variables and, due to an asymptotic approximation, to perform a set of highly informative evaluations with a low computational cost.

In this way, it is possible to avoid failures in the design strategy in the case of nonconvex problems and to apply an appropriate design procedure even when sufficient conditions are not met. As a result, one should be able to solve a larger class of problems and to assess the quality of the design by obtaining the actual probability of satisfying the design requirements.

Probabilistic Environment

An important aspect connected to a probabilistic formulation of the problem lies in the possibility of properly choosing the controller configuration within a set of different controllers that equally satisfy the same deterministic requirement. Because of the presence of uncertainties in the problem, the controllers do not satisfy the design requirement in the same way, but they do satisfy with a certain probability. For this reason, the best controller may be defined as the one which has the highest probability of satisfying the design requirements.

We start by defining the probability that the event of interest takes place as

$$P = \int_D f_X(\boldsymbol{x}, \boldsymbol{p})\, \mathrm{d}x \tag{1}$$

where $\boldsymbol{p} = (p_1, p_2, \ldots, p_m)^T$ is a deterministic parameter vector and $f_X(\boldsymbol{x})$ is the probability density function of the random variable vector $\boldsymbol{X} = (X_1, X_2, \ldots, X_n)^T$. Actually, P represents the probability of not complying with the design requirements. The domain of integration of $f_X(\boldsymbol{x})$ is $D = \{g(\boldsymbol{x}, \boldsymbol{p}) \le 0\}$, where $g(\boldsymbol{x}, \boldsymbol{p})$ is the limit state function having the properties that $g(\boldsymbol{x}, \boldsymbol{p}) > 0$ denotes the favorable states, $g(\boldsymbol{x}, \boldsymbol{p}) = 0$ the limit state, and $g(\boldsymbol{x}, \boldsymbol{p}) \le 0$ the unfavorable states. The domain region D is referred to as the admissible region. From a practical standpoint, Eq. (1) can be rather difficult to evaluate, especially when the uncertainty space is of high dimensions or when the limit state function is of a complex nature. For these reasons, analytical results are hard to find. A possible choice would be to execute Monte Carlo simulations, i.e., to solve Eq. (1) numerically. Another possibility is to use some of the efficient methods that have been developed in the last two decades in the field of structural reliability engineering to approximate the evaluation of P. These methods are referred to as first- or second-order reliability methods (FORM and SORM[2]).

In essence, these methods reduce the cumbersome task of integrating Eq. (1) to that of locating the most probable point in the admissible region. The idea is that of using a set of simple algebraic manipulations to transform the problem in such a way that an exact or asymptotic result may easily be obtained. The key result is this: if the variables X_i are jointly normal distributed and the limit state function is a hyperplane in the form

$$g(\boldsymbol{X}) = a_0 + \sum_{i=1}^{n} a_i X_i := \boldsymbol{a}_0 + \boldsymbol{a}^T X \tag{2}$$

then an analytical solution of Eq. (1) is given by

$$P = \Phi(-\beta) \tag{3}$$

where β is known as the reliability index and is defined by the equation

$$\beta := \frac{a_0 + \boldsymbol{a}^T E[\boldsymbol{X}]}{\sqrt{\boldsymbol{a}^T C_X \boldsymbol{a}}} \tag{4}$$

In Eq. (4), $E[\boldsymbol{X}]$ is the vector of the expected values, C_X is the covariance matrix of $\boldsymbol{X}$ and $\Phi(\cdot)$ is the standard normal probability distribution function. Note that if the basic variables X_i are independent standard normally distributed (in this case we will denote them by $\boldsymbol{U}$), β is still given by Eq. (4) with $E[\boldsymbol{U}] = 0$ and $C_U = I$, where I is the identity matrix.

Moreover, if the limit state function is not linear in $\boldsymbol{X}$, the problem may be approximated as[3]

$$P \approx \Phi(-\beta) \tag{5}$$

with

$$\beta = \|\boldsymbol{u}^*\| := \min\{\|\boldsymbol{u}\|\} \text{ for } \{\boldsymbol{u} \mid g(\boldsymbol{u}) \le 0\} \tag{6}$$

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where $\|\cdot\|$ denotes the Euclidean norm and $\boldsymbol{u}^*$ is referred to as β-point.

In general, the random variables X_i are not normally distributed. It has been shown[4,5] that this difficulty can be eliminated by introducing a suitable one-to-one transformation, say, T, of the basic random variables $\boldsymbol{X}$ into uncorrelated and standard normally distributed random variables $\boldsymbol{U}$ such that, on this transformed basis, the following result is asymptotically exact[6]:

$$P \sim \Phi(-\beta) \cdot \prod_{i=1}^{n-1} (1 - \kappa_i \beta)^{-\frac{1}{2}} = \Phi(-\beta) \cdot C \tag{7}$$

The symbol $\sim$ in Eq. (7) means that the calculation of P is exact in the limit when $\beta \rightarrow \infty$ (and, correspondingly, $P \rightarrow 0$). Also, C is a second-order correction. We stress the fact that the transformation T is always possible for continuous random variables with invertible distribution functions.

For practical purposes, Eq. (7) may be used whenever β is greater than one (which corresponds to $P < 0.841$), but, under some additional mild restrictions, it is possible to extend the result to the case where $|\beta| \leq 1$ (Ref. 7). A further extension of the theory was obtained for the case where the limit state function is given as the intersection of several individual state functions.[8] A situation with conditional probabilities has also been discussed.[9]

Design Philosophy

The design philosophy may be summarized as follows: given the values of certain parameters to be achieved by the controlled system, find a family of controllers, e.g., S, which nominally attain these values. The uncertainties of the parameters of the nominal structure are taken into account by modeling them as random variables with appropriate distribution functions. The optimal controller is that which belongs to S and minimizes the probability of not complying with the design requirements. The limit state function, i.e., the function $g(\boldsymbol{x}, \boldsymbol{p})$, which delimits the admissible region of the system, generally needs to be obtained through a response surface approach.[10] The reason is that an analytical closed form solution is usually unavailable for real engineering problems. The response surface approach can be used to relate the parameters-of-interest of the closed loop system with those parameters of the open loop system that are known with uncertainties. The computational procedure is described in nine steps:

1) Define the values that the objective function-of-interest should achieve.

2) Find a family of controllers that nominally attain these values and satisfy the requirements to be met by the structure. Call such a family S.

3) Model the parameters that are known with uncertainty as random variables with appropriate probability distribution functions.

4) For all the controllers belonging to S, execute a $3 - k$ factorial experiment relating the value of the objective function to the parameters modeled as random variables.

5) Obtain a response surface from the data obtained, and check the error distribution through Monte Carlo simulations.

6) Define the limit state function by using the information of the previous steps.

7) Transform the problem from the physical space to a normalized space (i.e., transform the random variables to independent normal distributed random variables) and, accordingly, transform the limit state function.

8) Compute the probability that the desired value of the objective function of the controlled system is higher (lower) than the design bound.

9) Identify the most appropriate controller (within the family S of controllers) as the one that has the better behavior with regard to the design goal in a probabilistic sense.

With such an approach, we obtain an optimum in a probabilistic sense within the controllers of the family S even if the design procedure does not evolve automatically up to the best controller, i.e., the controller that has the highest probability of satisfying the design constraints. Nevertheless, the proposed procedure is appealing, not only for the quality of information one may obtain, but also because the usage-of-parameters study may be employed to address the evolution of the design in the direction of a better probability value.

Numerical Example: A Gust-Alleviation System

As an example, we consider the design of a gust-alleviation control system for a transport aircraft. The problem is to reduce the number of crossings of a certain load level at a fixed aircraft location: this reduction can be related to the reduction of the fatigue damage on that point of the aircraft. The approach is as follows: first, recall that the number of times, $N(y)$, that a certain load level y is exceeded for a given aircraft station can be derived by using the Rice formula,[11] which reads

$$N(y) = N_0 \exp\left[-\frac{1}{2}\frac{(y/\bar{A})^2}{\sigma_w^2}\right] \tag{8}$$

where

$$\bar{A} := \sigma_y / \sigma_w \tag{9}$$

σ_y and σ_w being the root mean square values of the load and of the gust velocity, respectively, and N_0 is the number of crossings of a given level y, per unit time, with positive slope. Also, it is possible to obtain N_0 by means of the following equation[11]:

$$N_0 = \frac{1}{2\pi}\frac{\sigma_{\mathrm{d}y/\mathrm{d}t}}{\sigma_y} \tag{10}$$

where, obviously, $\sigma_{\mathrm{d}y/\mathrm{d}t}$ is the root mean square value of the variation of the load level with time. From Eq. (8), one can identify an appropriate control law whose aim is the reduction of N_0. For a given gust loading level, i.e., σ_w, the bounds on N_0 define the intensity of the stresses on the aircraft. Various deterministic controllers can be obtained that achieve the same value of N_0 reduction. In this example, we used a state space model of a transport aircraft reported in the literature[12] where the first three elastic modes have been retained (in addition to the rigid short-period modes). A Dryden filter was used to model the effects of a vertical gust on the aircraft.

The problem can then be stated as follows:

1) Fix the desired reduction level of N_0, say $N_{0\,\mathrm{des}}$.

2) Define the objective function f to be minimized as

$$f = (N_0 - N_{0\,\mathrm{des}})^2 \tag{11}$$

3) Define the constraints on aircraft dynamics by means of the flight qualities requirements. These requirements may be expressed as

$$\omega_{\min} \leq \omega_{\mathrm{sp}} \leq \omega_{\max}, \qquad \zeta_{\mathrm{sp}} \geq \zeta_{\mathrm{sp}_{\min}} \tag{12}$$

ω_{sp} and ζ_{sp} being, respectively, the short-period frequency and the damping.

The problem has been numerically solved by means of a static output feedback controller with gain matrix K in the following way:

1) Choose a control gain matrix, K.

2) Evaluate the closed loop state matrix.

3) Calculate the values of $\sigma_{\mathrm{d}y/\mathrm{d}t}$, of σ_y, and of N_0.

4) Through an optimization procedure, the elements of the matrix K are iteratively varied until the objective function f [see Eq. (11)] is minimized.

The problem presents multiple solutions, i.e., multiple matrices $K \in S$ (S being the set of feasible solutions), which are equivalent in a deterministic framework. This equivalency is not true when the uncertainties in the aerodynamic derivatives (which implies uncertainties in terms of the system state matrix) are taken into account. The satisfaction of the design requirements is no more equivalent if a probabilistic evaluation is carried on, because the various controllers have a different probability of satisfying the design requirements: the "best" controller is the one which has the highest probability of satisfying such constraints.

In particular, in this example, the aerodynamic derivatives Z_w, M_w, and M_q have been considered as random variables with rectangular distribution centered around the nominal value and with an excursion of 10% about such a value.

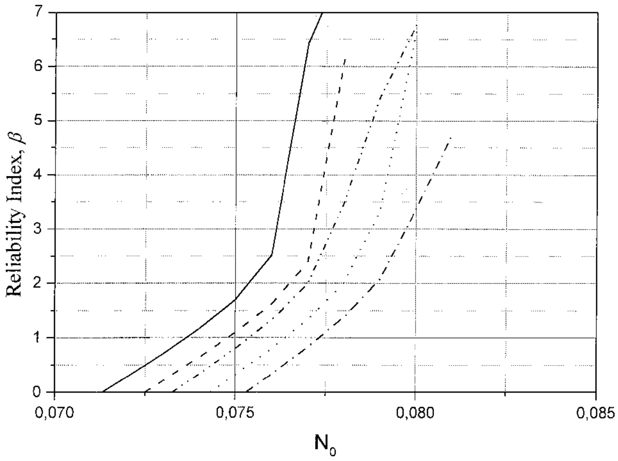

Fig. 1 Comparison of different controllers in terms of probability of not satisfying the project requirements.

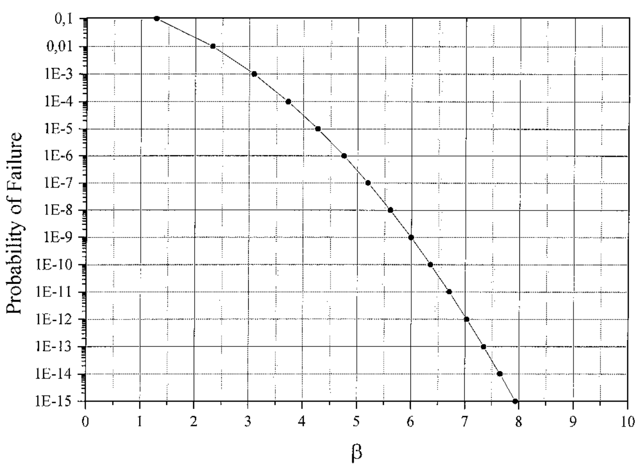

Fig. 2 Relation between the reliability index and the probability of failure.

Obviously, the optimal controller is the one which belongs to S and which minimizes the probability of not complying with the design requirements. A response surface approach was used to relate the values of the aerodynamic derivatives, which are known with uncertainties, to the corresponding value of N_0.

In Fig. 1, the results of the probabilistic analysis are reported in terms of the reliability index β, where the different behavior of the deterministically equivalent controllers may be compared. The relationship between β and the cumulative probability is reported in Fig. 2 for completeness. In particular, Fig. 1 shows the probability of reducing N_0 for the controlled system. The open loop value of N_0 is 0.0874, consequently, all the reported controllers should bring the value of N_0 below approximately 0.07 to comply with the design requirement. As can be seen, different behaviors are obtained for different controllers even though the same N_0 reduction has been achieved in the deterministic case. Obviously, the best controller in the set is the one represented by a continous line. Recalling that N_0 is strictly related to the stresses on the aircraft, this reduction should result in a different behavior of the structure with respect to fatigue performance and, accordingly, should affect aircraft inspection strategies.

Conclusions

The aim of this Note was to study the effects of parameter uncertainties in the design of a control system in order to properly define the design with respect to the satisfaction of the design constraints that the controlled structure should meet. A case study was presented which clearly shows how the use of a probabilistic analysis may help the engineer in evaluating the uncertainties that are inherently present in almost every design. Structural reliability methodologies were used to constitute the probabilistic framework to perform the analysis. The results appear to be promising for obtaining a more rational guideline for project development by properly taking into account the uncertainties in the design variables.

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Temperature Measurements in a Hypersonic Boundary Layer Using Planar Laser-Induced Fluorescence

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Introduction

THE boundary layer flow over hypersonic vehicles plays a significant role in determining the performance of engine inlets, lifting surfaces, and control flaps. It is therefore critical to have a detailed understanding of relatively simple boundary-layer flows, such as that which develops over a flat plate, to assess possible design input limitations for these systems. One characteristic of hypersonic

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